9 The Minimal Supersymmetric Standard Model: One-Loop

9.1 Gauge Coupling Unification

$$g_1 \equiv \sqrt{\frac{5}{3}} g' = \sqrt{\frac{5}{3}} \frac{e}{\cos\theta_W} \tag{9.1}$$

$$g_2 \equiv g = \frac{e}{\sin\theta_W} \tag{9.2}$$

$$g_3 \equiv g_s \tag{9.3}$$

$$\alpha_a \equiv g_a^2/4\pi \tag{9.4}$$

These couplings run:

$$\frac{\mu d}{d\mu} g_a = -\frac{1}{16\pi^2} b_a g_a^3 \qquad \Rightarrow \qquad \frac{\mu d}{d\mu} \alpha_a^{-1} = \frac{b_a}{2\pi} \tag{9.5}$$

in the Standard Model (including the top quark)

$$b_a^{\rm SM} = (-41/10, 19/6, 7) \tag{9.6}$$

while in the MSSM

$$b_a^{\rm MSSM} = (-33/5, -1, 3). \tag{9.7}$$

In the MSSM with a common threshold $M_{\rm SUSY}$ for the superpartners the couplings appear to unify at a scale $M_U \approx 2 \times 10^{16}$ GeV.

This is intriguing, but one should keep in mind that we are solving three equations in three unknowns: M_U, $\alpha(M_U)$, and $M_{\rm SUSY}$. So we are garaunteed a solution. Thus the statement is that it is interesting that the solution occurs for a reasonable value of $M_{\rm SUSY}$. Taking into account various uncertainties [5] one finds solutions in the range:

$$3\,\mathrm{GeV} < M_{\rm SUSY} < 100\,\mathrm{TeV}. \tag{9.8}$$

There are additional uncertainties due to thresholds at superpartner masses and M_U.

Figure 1: Running gauge couplings in the SM and MSSM [1]. In the MSSM $\alpha_3(m_z)$ is varied between 0.113 and 0.123, and M_{SUSY} between 250 GeV and 1 TeV.

9.2 Radiative Electroweak Symmetry Breaking

RG equations for soft masses:

$$\begin{aligned} 16\pi^2 \frac{d}{dt} m_{H_u}^2 &= 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2, && (9.9)\\ 16\pi^2 \frac{d}{dt} m_{H_d}^2 &= 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2. && (9.10) \end{aligned}$$

$$\begin{aligned} 16\pi^2 \frac{d}{dt} m_{Q_3}^2 &= X_t + X_b - \frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2 && (9.11)\\ 16\pi^2 \frac{d}{dt} m_{\bar{u}_3}^2 &= 2X_t - \frac{32}{3}g_3^2|M_3|^2 + -\frac{32}{15}g_1^2|M_1|^2 && (9.12)\\ 16\pi^2 \frac{d}{dt} m_{\bar{d}_3}^2 &= 2X_b - \frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 && (9.13)\\ 16\pi^2 \frac{d}{dt} m_{L_3}^2 &= X_\tau - 6g_2^2|M_2|^2 - \frac{3}{5}g_1^2|M_1|^2 && (9.14)\\ 16\pi^2 \frac{d}{dt} m_{\bar{e}_3}^2 &= 2X_\tau - \frac{24}{5}g_1^2|M_1|^2. && (9.15) \end{aligned}$$

where

$$\begin{aligned} X_t &= 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2, && (9.16)\\ X_b &= 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\bar{d}_3}^2) + 2|a_b|^2, && (9.17)\\ X_\tau &= 2|y_\tau|^2(m_{H_d}^2 + m_{L_3}^2 + m_{\bar{e}_3}^2) + 2|a_\tau|^2. && (9.18) \end{aligned}$$

Note that in running to the IR, squark (mass)^2 are driven positive by $|M_3|^2$ terms.

Gauge terms are additive so we can consider them seperately. Keeping on the $|y_t|^2$ terms we have

$$16\pi^2 \frac{d}{dt} m^2_{H_d} = 0 \tag{9.19}$$

$$16\pi^2 \frac{d}{dt} \begin{pmatrix} m^2_{H_u} \\ m^2_{\overline{u}_3} \\ m^2_{Q_3} \end{pmatrix} = 2|y_t|^2 \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} m^2_{H_u} \\ m^2_{\overline{u}_3} \\ m^2_{Q_3} \end{pmatrix} \tag{9.20}$$

Starting with $m^2_{H_u}$, $m^2_{\overline{u}_3}$, and $m^2_{Q_3}$ all equal to m_0^2 at some high scale, we see that the masses run to

$$\frac{m_0^2}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \tag{9.21}$$

So $m^2_{H_u}$ runs negative. It is usually claimed that this garauntees EWSB, but as we have seen EWSB may or may not follow depending on the values of mu and b. It is also usually claimed that this calculation "predicted" a large top mass, but it really only required that

$$y_t = \frac{\sqrt{2}\, m_t}{\sin\beta} \tag{9.22}$$

was large.

9.3 One-loop correction to the Higgs mass

Recall

$$m_h < |\cos 2\beta| m_Z = \frac{g^2 + g'^2}{4} |v_d^2 - v_u^2| \tag{9.23}$$

The Higgs mass is controlled by the quartic couplings. As we have seen below a SUSY violating threshold, the quartic couplings run independently of the gauge couplings. If the stop squarks are heavy compared to the top then

$$\lambda(m_t) = \lambda_{\mathrm{SUSY}} + \frac{4N_c|y_t|^4}{16\pi^2} \ln\left(\frac{m_{\widetilde{t}}}{m_t}\right) \tag{9.24}$$

$$
\begin{aligned}
\Delta(m_{h^0}^2) &= \frac{3}{2\pi^2} v^2 y_t^4 \sin^2\beta \, \ln\left(\frac{m_{\tilde{t}}}{m_t}\right) \\
&\approx \frac{(90\,\mathrm{GeV})^2}{\sin^2\beta} \qquad (9.25)
\end{aligned}
$$

Note

$$
\frac{d}{dt} y_t = \frac{y_t}{16\pi^2}\left[6|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right] \qquad (9.26)
$$

With the additional assumption that y_t does not blow up below the unification scale we can put a lower bound on $\sin\beta$. With this bound and some other smaller corrections one finds a one-loop radiative correction to the Higgs mass bound:

$$
m_{h^0} < 130\,\mathrm{GeV} \qquad (9.27)
$$

Adding new singlet fields gives:

$$
m_{h^0} < 150\,\mathrm{GeV}. \qquad (9.28)
$$

9.4 Precision Electroweak Measurements

Below the EWSB scale we can have terms in the effective Lagrangian like

$$
\mathcal{L}_{\mathrm{eff}} \subset -\frac{S\,gg'}{16\pi} W^3_{\mu\nu} B^{\mu\nu} \qquad (9.29)
$$

A heavy fermion (like the top) that gets a mass from EWSB contributes to the W^3-B vacuum polarization $\Pi^{3B}_{\mu\nu}(p^2)$ Since

$$
Tr T^3 Y_L = 0 \qquad (9.30)
$$

$$
Tr T^3 Y_R = Tr T^3 Q = \frac{1}{2} \qquad (9.31)
$$

$\Pi^{3B}_{\mu\nu}(p^2)$ is proportional to $p^2 g_{\mu\nu} - p_\mu p_\nu)$

$$
\Pi^{3B}_{\mu\nu}(p^2) = m^2 \int d^k \, F_{\mu\nu}(p, k, m) \qquad (9.32)
$$

For $m \gg m_Z$

$$
\frac{d}{dp^2}\Pi^{3B}_{\mu\nu}(p^2)|_{p^2=0} \propto \frac{m^2}{m^2} \qquad (9.33)
$$

We see non-decoupling as $m \to \infty$, because $m \propto v$.

For a superpartner mass we have $m_{\mathrm{sp}}(m_{\mathrm{soft}}, \mu, v)$. In the limit μ, $m_{\mathrm{soft}} \to \infty$ with v fixed we have $m_{\mathrm{sp}} \to \infty$, so radiative corrections to S and related parameters go like $v^2/m_{\mathrm{sp}} \to 0$. Thus the superpartners decouple from EWSB if they are sufficiently heavy.

9.5 Problems with Flavor and CP

At generic points in the 105 dimensional parameter space there are flavor changing and CP violating effects that contradict experiment. fopr example lepton number can be violated

$$\Gamma_{\mu\to e\gamma} \approx 4sin^2\theta_w \left(\frac{\alpha_2}{4\pi}\right)^3 \frac{m_\mu^5}{M_{\rm SUSY}^4} \left(\frac{\Delta m_L^2}{M_{\rm SUSY}^2}\right)^2 \tag{9.34}$$

$$\Gamma_\mu = \left(\frac{\alpha_2}{4\pi}\right)^2 \frac{\pi m_\mu^5}{64 m_W^4} \tag{9.35}$$

$$\frac{\Gamma_{\mu\to e\gamma}}{\Gamma_\mu} \approx 4\times 10^{-5} \left(\frac{500\rm GeV}{M_{\rm SUSY}}\right)^4 \left(\frac{\Delta m_L^2}{M_{\rm SUSY}^2}\right)^2 \tag{9.36}$$

Experimentally this ratio is less than 5 $\times 10^{-11}$.

In the standard model the $K\overline{K}$ mixing amplitude is

$$\mathcal{M}^{SM} \approx \alpha_2^2 \frac{m_c^2}{m_W^4} \sin^2\theta_c cos^2\theta_c \tag{9.37}$$

In the MSSM we have contributions like

$$\mathcal{M}^{MSSM} \approx 4\alpha_3^2 \left(\frac{\Delta m_Q^2}{M_{\rm SUSY}^2}\right)^2 \frac{1}{M_{\rm SUSY}^2} \tag{9.38}$$

For $\mathcal{M}^{SM} < \mathcal{M}^{MSSM}$

$$\left(\frac{\Delta m_Q^2}{M_{\rm SUSY}^2}\right)^2 < 4\times 10^{-3} \frac{M_{\rm SUSY}}{500\,\rm GeV} \tag{9.39}$$

A terms also introduce off-diagonal mixing. For example the electric dipole moment of the d quark is apprriximately

$$\frac{\alpha_3}{4\pi} \frac{evc_\beta a_d \delta}{M_{\rm SUSY}^2} \tag{9.40}$$

while the electic dipole moment of the neutron is $< 0.97\times 10^{-25}$ e cm. so

$$c_\beta a_d \delta \left(\frac{500GeV}{M_{\rm SUSY}^2}\right)^2 < 5\times 10^{-7} \tag{9.41}$$

For $a_d = y_d$

$$\delta \left(\frac{500GeV}{M^2_{\rm SUSY}} \right)^2 < 10^{-2} \tag{9.42}$$

To avoid these problems we need to be in “safe neighborhoods” of the parameter space. Three safe neighborhoods that have been identified are

- 1) “Soft Breaking Universality” there are three conditions

  $$\mathbf{m^2_P} \propto \mathbf{I} \tag{9.43}$$
  $$\mathbf{A_P} \propto \mathbf{Y_P} \tag{9.44}$$

  and no new non-trivial phases

- 2) “More Minimal Supersymmetric Model”[6] only require leading quadratic divergences to cancel: $\widetilde{t}_L$, $\widetilde{t}_R$, $\widetilde{b}_L$, $\widetilde{H}_u$, $\widetilde{H}_d$, $\widetilde{B}$,$\widetilde{W}$ have masses below 1 TeV, while first and second generation sparticle can be as heavy as 20 TeV. However two-loop running below the heavy squark threshold

  $$\frac{dm^2_{\widetilde{t}}}{dt} = \frac{8g_3^2}{16\pi^2} C_2 \left[\frac{3g_3^2}{16\pi^2} m^2_{\widetilde{u},\widetilde{d}} - M_3^2 \right], \tag{9.45}$$

  may drive the top squark mass^2 negative.

- 3) “Alignment” [7, 8]

  $$\mathbf{m^2_Q} = \mathbf{Y^*_u Y^T_u} + \mathbf{Y^*_d Y^T_d} \tag{9.46}$$
  $$\mathbf{m^2_u} = \mathbf{Y^\dagger_u Y_u} \tag{9.47}$$
  $$\mathbf{m^2_d} = \mathbf{Y^\dagger_d Y_d} \tag{9.48}$$

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